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ABSTRACT

This report defines spatial ability operationally (consisting of three components) and reviews possible sex differences in this ability as reported in the literature. The relationship between mathematical ability and spatial ability is discussed. Researchable hypotheses about how differential spatial ability may effect mathematics achievement are suggested. (Author/LS)

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MATHEMATICS, SPATIAL ABILITY AND THE SEXES

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In writing a paper about mathematics and spatial ability, one is confronted with two dilemmas: (1) there is too much information on spatial ability to review the literature adequately, and (2) there is too little information about spatial ability and mathematics to come to any final conclusion about their relationship. In spite of these dilemmas however, this paper will try to give an overview of sex differences in spatial ability and how mathematics and spatial ability might be related. Also some suggestions for researchable hypotheses about how differential spatial ability may effect mathematics achievement will be made.

Spatial Ability and the Sexes

Male superiority over females in tasks measuring spatial ability is an accepted truism which has been documented by many authors (Garai and Scheinfeld, 1968; Maccoby, 1966; Kagan and Kogan, 1970; Kogan, 1972; Sherman, 1967; Anastasi, 1958; Tyler, 1956). Maccoby and Jacklin recently (1973) confirmed this again when they summarized the literature by stating that "spatial ability continues to be the area (i.e. intellectual area) in which the strongest and most consistent sex differences are found." When significant differences in performance on spatial tasks are found, they usually indicate boys' superiority. However, such differences are often of relatively

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small magnitude and many times the variation within sexes is greater than the variation between sexes. This is particularly true in the area of the field independence tasks* reported by Witkin (Kagan and Kogan, 1970). This sex difference in performance on spatial tasks has received considerably more attention than sex differences in mathematical ability even though some believe that the difference in spatial ability is much less pronounced and consistent than the sex differences in mathematical ability (Kogan, 1972).

When sex differences in spatial task performances appear is unclear. Coates (1973) in a review of field independence studies, concluded that a majority of the studies, where a valid measure of field independence was used, indicate that girls perform at significantly higher levels than do boys before the age of five. In early summaries Maccoby (1966) believed differences in spatial ability favoring males were evident by early school years while in a recent publication (Maccoby and Jacklin, 1973) the conclusion was that "such differences are minimal and inconsistent until 10-11 years of age when the superiority of boys becomes consistent."

It is interesting to note that this sex difference in performance on spatial tasks does not appear in all cultures. Kabanova-Meller (1970) reports that sex differences do not exist between Russian boys and girls in Grades 4, 5 and 6, although as is usual

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*These field independence tasks have been shown to be spatial in character (Sherman, 1967; Sherman, in press; Maccoby and Jacklin, in press;) and properly belong in a discussion of sex differences in spatial task performance.

in the Russian literature, little empirical data was reported to substantiate this belief. Berry (1966) and Kleinfeld (1973) report that while Eskimos appear to have highly developed spatial skills, no difference is found between spatial ability of male and female Eskimos.

When significant sex differences in spatial ability in our culture appear and, although the magnitude of the difference may be small, there seems to be concensus that there is male superiority in performance on spatial tasks, evident at least slightly before or at the onset of puberty and continuing well into adulthood. Of particular importance to this discussion is the paralleling of development of sex differences in mathematics achievement and spatial ability. No significant sex differences in either mathematics achievement nor spatial task performance have been consistently reported in subjects of 4-8 years of age. Sex differences in performance on both types of tasks become more pronounced between upper elementary years and the last year of high school and the differences show a pronounced increase during this time-span (Fennema, 1974; Maccoby and Jacklin, 1973).

It appears reasonable to hypothesize that since there is this concurrent developmental trend and since tests of spatial ability contain many of the same elements contained in mathematics, the two abilities might interact and affect the learning of mathematics. Perhaps less adequate spatial ability may partially explain girls' inferior performance in mathematics.

The Relationship Between Mathematics and Spatial Ability

Defining and describing spatial ability has been the job of complete books as well as chapters in other books (Smith, 1964; Kagan and Kogan, 1970; Werdelin, 1961). It appears not to be a unitary factor but one made up of two or more factors, the number and names of these factors differing with various authors (Smith, 1964). One factor, which is generally agreed upon is spatial visualization which involves visual imagery of objects and movement or change in the objects themselves or change in their properties. In mathematical terms, spatial visualization requires that objects be rotated, reflected and/or translated. Particularly in the area of spatial visualization is the relationship between mathematics and spatial ability logically evident.

One area of mathematics which is closely related to spatial visualization is Geometry. James and James (1968, p. 162) define geometry as "the science that treats of the shape and size of things... The study of invariant properties of given elements under specified groups of transformations". A geometrician could easily look at most spatial visualization tests and recognize transformational geometry ideas represented pictorially. Many mathematicians believe that all of mathematical thought involves geometrical ideas. The total discipline of mathematics has been defined as the language for describing those aspects of the world which can be stated in terms of "configurations" (Bronowski, 1947). Meserve (1973, p. 249) believes that each person who makes extensive use of all areas of

mathematics uses the modes of thought of geometry at every turn and that "even the most abstract geometrical thinking must retain some link, however attenuated, with spatial intuition." In the Russian literature, Mathematics and spatial abilities are regarded as inseparable (Kabanova-Meller, 1970).

Not only are spatial components an integral part of the structure of mathematics, but spatial representations are being increasingly included in the teaching of mathematics. E.g. the Piagetian conservation tasks, which are becoming a part of many preschool programs, involve focusing on correct spatial attributes before quantity, length, and volume are conserved. Most concrete and pictorial representations of arithmetical, geometrical and algebraic ideas appear to be heavily reliant on spatial attributes. The number line, which is used extensively to represent whole numbers and operations on them, is a spatial representation.

Even focusing on the important mathematical attributes of counters involves being able to spatially separate each counter from its environment and to see it as a distinct entity. This must require at least an ability similar to the one required to do the tasks on the Embedded Figures Test - a common test of spatial ability. Commutativity of multiplication illustrated by turning an array 90 degrees, involves a direct spatial visualization skill. Many other examples could be cited.

Although the relationship of mathematical ability and spatial ability appears logical, empirical data confirming a positive

relationship is less clear. Many factor analytic studies have explored this relationship and several authors have reviewed the literature. Some investigators have definitely concluded that spatial ability and mathematics ability are not related. In 1967, Very (p. 172) concluded "Research on spatial ability has failed to produce any significant correlation of (the spatial factor) with any facet of mathematics performance." Fruchter (1954, p. 2) stated that "spatial ability is unrelated to academic performance with the possible exception of a few very specialized courses such as engineering drawing." Smith (1964, p. 127, p. 68) concluded that although "there are several studies which indicate consistently that spatial ability is important in tests which are genuinely mathematical as distinct from those which involve purely mechanical or computational processes . . . the question whether the mathematical ability is dependent on the visual factor (or factors) has not been definitely answered."

Even in the specialized mathematical area of geometry where logically one would expect to find the strongest relationship, empirical findings do not clearly indicate that the two are related. Lim concluded in 1962 after a thorough review of relevant literature "Unfortunately the evidence for a relationship between geometric ability and spatial visualization remains inconsistent and unreliable." Werdelin (1961, p. 39) was also not willing to conlcude definitely that empirical data indicated that spatial ability and geometry ability were related. However, he felt that "there is strong pedagogical

reason to believe in a connection between the ability to visualize and geometric ability."

Other authors feel that data indicate a positive relationship. In 1951, Guilford, Green and Christensen conlcuded that spatial visualization ability helped in solving mathematics problems. French (1951, 1955) also showed that successful achievement in mathematics depends to some extent on use of spatial visualization skills. In a recent review Aiken (1973, p. 406-7) concluded that spatial-perceptual ability was one of the "most salient" mathematical factors extracted in various investigations.

Obviously, the relationship between learning in mathematics and spatial ability is not clear and the need for more data is great. Even less is known about the effect that differential spatial ability has on the mathematics learning of boys and girls. Several questions come to mind which, when investigated may provide some important insight into the relationship between mathematics learning, spatial ability and the learning of mathematics by boys and girls.

What is the effect of spatial ability on mathematical learning at various developmental levels?

Smith (1964) has hypothesized that while spatial ability may not be related to mathematics ability at beginning stages of mathematics learning, advanced mathematics learning increasingly depends upon spatial ability. It would appear that this hypothesis was made after surveying a number of studies which used high school or col-

lege students as subjects and relatively sophisticated mathematical ideas as criterion measures. Little or no data were presented from studies with younger learners. However, in 1964 one could have built a strong argument that logically supported the idea that spatial ability was not related to mathematics ability at beginning stages of mathematics learning. Little or no geometry was taught at the pre-high school level and most pre-high school mathematics tests would not have included geometrical items. Such tests would have focused primarily on arithmetical/computational ideas, the latter of which has been found to be negatively correlated with spatial ability (Werdelin, 1958). Therefore, Smith's hypothesis that spatial ability was not related to mathematical ability at beginning stages of mathematics learning was logical in 1964 because the tests used to measure mathematics achievement probably included few items relevant to spatial ability. These tests reflected the mathematics program previous to 1964.

However, since 1964, a major change has taken place in most K-12 mathematics curricula. More emphasis is placed on the structure of mathematics and its underlying principles. Geometry has become an integral past of the entire mathematics curriculum. Increasingly mathematics is taught as an interrelated system of ideas. In order to learn new ideas, learners are dependent upon prerequisite ideas in their cognitive structure. Little is known concerning the impact of spatial ability on the acquiring of these prerequisite mathematical ideas on which all later mathematical knowledge is based. It appears to be of the utmost importance.

Developmental psychologists patterned on Piaget have theorized that at different stages of cognitive development certain modes of thought predominate and ideas are added to one's cognitive structure by utilizing actions, symbols which represent those actions, and symbols along in somewhat different blends. According to this theory, mental structures are formed by a continual process of accommodation to and assimilation of the environment. This adaptation (accommodation and assimilation) is possible because of the actions performed by the individual upon her environment. These actions change in character and progress from overt, sensory actions done almost completely outside the individual; to partially internalized actions which can be done with symbols representing previous actions; to complete abstract thought done entirely with symbols. Thus, development in cognitive growth progresses from the use of physical actions to form schemas to the use of symbols to form schemas, i.e., learners change from a predominant reliance upon physical actions to a predominant reliance upon symbols.

Mathematical educators have increasingly accepted this theory of cognitive development and have translated it into educational practice by an increased emphasis upon the instructional use of three modes to represent mathematical ideas, i.e., concrete (enactive), pictorial (ikonic), and symbolic. The blend of the usage of these representational modes should reflect the cognitive developmental level of the learner. Particularly at early stages of mathematical

learning it is important to provide learners with concrete representations of mathematical ideas while symbols assume increasing importance as learners mature and mathematical ideas become more complex.

As was illustrated earlier, most concrete and pictorial representations of mathematical ideas include spatial attributes, some of which are relevant to the mathematical idea being taught and some of which are not. Since the only way to add simple mathematical ideas to one's cognitive structure at early developmental levels is by interaction with concrete or pictorial materials which represent those ideas, and since those representations depend heavily on spatial attributes, if for some reason one is hampered in perception of those spatial attributes then one is hampered in learning those early mathematical ideas. Without knowledge of these ideas, it is impossible to learn advanced mathematics. Therefore, spatial ability or the ability to learn spatially appears to be of utmost importance at early stages of learning.

Sherman (1967) has suggested that boys outperform girls on spatial tasks because they participate voluntarily in more spatially oriented activities. Girls learn to read more easily than do boys. Because of ease of use of symbols i.e., reading, do girls voluntarily or are they encouraged to rely more heavily on symbols to learn mathematics rather than using concrete or pictorial representations? If so, perhaps inadequate usage of spatial representations may hamper both the development of their spatial ability and ability to do well in more advanced mathematical learning.

No data is available to give insight into this question. Empirical data from studies dealing with the use of various representational modes are not conclusive about even the value of concrete and pictorial representations and as far as this author knows no study has included spatial ability as a control factor. Certainly, more data are needed.

What is the interaction effect of other abilities and spatial ability on achievement in mathematics?

Werdelin (1961) showed that girls were able to prove verbal theorems better than boys but were less able to translate words into figural images and then to transform those images in a directed way. It is an accepted truism also that females' verbal ability is more highly developed than males. Does the development of verbal ability in some way interfere with development of spatial ability? Werdelin (1958) in a factor analytic study found one visual factor in high school students which was related to a factor he called a mathematical reasoning factor. Interestingly, he found the correlation between his visual factor and a numerical (or computational) factor was negative. Females often score higher on tests of computation than do males. Perhaps higher development of numerical or computational ability interfered with development of spatial ability. Both of these questions appear related to the earlier one of the impact of spatial ability on early mathematical learning. Does facility with symbols--computational or verbal-- interfere with development of spatial ability?

What is the effect of possessing a greater variety of well developed abilities on mathematics learning?

Harris and Harris (1972), Werdelin (1958), and Very (1967) have shown a larger number of space factors for males than for females. Werdelin (1961) concluded that if one could attack a problem either verbally or spatially, one would be more apt to be able to solve it, as his data showed that boys were superior on items which measured the ability to comprehend the organization of a visual figure and to reorganize. Where items could be solved by verbal means and did not require that the problems be translated into a mental figure, no sex differences were found. Perhaps because males have developed more abilities than have females they are enabled to attack mathematical problems in a variety of ways and thus are able to score higher on mathematical achievement tests.

What sex differences in mathematics achievement would be found if spatial ability were not a factor?

Tittle (1973) has shown that many tests, commonly used to measure achievement, are sexually biased. Certainly if a mathematics test contains many items that require spatial ability to solve, girls will not do as well as will boys. It would be very interesting to construct a test that had little or no spatial content in it and compare the sexes on achievement. Perhaps no differences will be found if the test content is controlled in the spatial area. On the other hand, spatial visualization may be such an integral part of higher mathematical thinking that eliminating spatial aspects of

mathematics tests too narrowly restricts the area of mathematical thinking. This aspect should be investigated.

Implications and Directions for Further Research

Data related to the questions found in this paper will be helpful. Also needed are data which give insight into how spatial ability is developed. Several investigators (Kleinfeld, 1973) have suggested that spatial ability is as important as are other abilities which have received extensive attention in the schools, i.e. verbal ability. Certainly a plethora of abilities will be more effective in dealing with modern day society than will one, so this appears reasonable. Although the main concern of this paper was to explore one facet of why girls achieve at lower mathematical levels than do boys, it is also hoped that one of the outcomes will be an increased awareness of a specialized ability that has received inadequate attention from mathematics educators in recent years. Hopefully, more data will be forthcoming in this important area.

It is tempting to look for a simplistic explanation for sex differences in mathematical achievement. If one says that such differences are the result of differences in spatial ability, one has such a simplistic explanation which is totally inadequate. This paper by no means suggests that sex differences in spatial visualization is the only factor contributing to sex differences in mathematics achievement. Other possible factors include the hypothesis

that fewer females than males inherit a gene for quantitative reasoning (Stafford, 1972); stereotyping of mathematics as a male domain; lack of encouragement of females by parents and peers; and lack of clearly perceived vocational plans for females which would include the use of mathematics. Nonetheless, spatial ability is one factor which may contribute to mathematics achievement and certainly this relationship warrants further investigation.

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